The compulsory education ends with the ordinary common schools, and the graduates of these schools are admitted to middle schools, though not infrequently an examination is required on account of the large number of applicants. The higher common school is intended for those who have to complete their education here. Its course regularly consists of two years, but it may extend to three years. The arithmetic in this school is roughly as follows:

First year: Greatest common measure, least common multiple, calculations with fractions in a more general sense than in the ordinary course, problems of application, problems in rates, simple and inverse proportions, and percentage partition.

Second year: Proportion, including compound proportion; drill in review of previous work. Geometrical drawing is also taught.

In the *third year*, when given, some further lessons in arithmetic are provided.

The middle schools extend through five years, the graduates of ordinary common schools being admitted. The lessons in mathematics are distributed somewhat as in the following table:

	1st Year	2nd Year	3rd Year	4th Year	5th Year
Arithmetic	3 3 3	3 3			
Algebra		3	2 2 2	2 2 2	2
Geometry			3 3 3	3 3 3	2 2 2
Trigonometry					2 2

The figures indicate the number of hours per week and the dotted lines indicate the divisions of terms, one school year consisting of three terms.

It is a matter of course that the distribution of lessons varies according to the different schools. The above is only a specimen.

In arithmetic, the following subjects are taught: Calculation of whole numbers, properties of the same, calculation of fractions, decimals, measures, money, etc. Simple, inverse, and compound proportions, problems in rates, percentage, profit and loss, interest.

The algebra course is somewhat similar to that given in the briefer of

Charles Smith's two treatises; including the four operations, indices, linear operations in one unknown, linear simultaneous equations, fractions, quadratic equations, irrational equations, extraction of square and cube roots, proportion, arithmetic and geometric progressions, interest, etc.

The geometry course is modeled after the treatise published by the Association for the Improvement of Geometrical Teaching in England. Proportion is treated according to the algebraic method. Solid geometry follows closely the outline of geometry in Wilson's treatise.

The trigonometry course follows closely the contents of John Casey's little work.

The text-books of the middle school are not prepared by the State as in the case of the common school, but are left to private authors. Nevertheless they are examined by the Department of Education, before they are authorized. There is a considerable number of these text-books now in use in Japan. Among those the most popuular are D. Kikuchi's Geometry, R. Fujisawa's Arithmetic, the same author's Algebra, etc. We are greatly indebted to the authors of these able treatises for the success of mathematics teaching in our country.

In the Japanese middle school all the boys are taught in the same manner, even though they may differ widely in their aims. Those who have to complete their education in the school and those who are preparing for higher grades of education are all instructed in the some school and all in the srme manner. Even the literary and scientific courses are not distinguished. Consequently the number of subjects taught becomes exceedingly large. And, moreover, the interrelations between these numerous subjects receive little attention. Even the different branches of mathematics are taught quite independently and irrespective of other branches. We hope however that such a defect as this will be remedied in the future, as a new program for the work of the schools is being prepared by the Department of Education, which will be made public very soon.

Before the graduates of the middle school may enter the university they must go through another school, whose course lasts three years. This school and other higher professional schools are all open to the graduates of middle schools, but as there is an insufficient number of these schools the candidates are obliged to pass competitive examinations, which proves a great barrier for young men. On this account they are naturally induced to pay most attention to the results of their examinations and the standing of these schools is determined solely by this criterion. It results therefore that the lessons are practically regulated with the sole aim of preparing for these examinations, which have thus acquired such an importance that they directly influence the teaching of the middle schools. It is consequently necessary to say something about these examinotions. As specimens we quote the problems given last summer in the entrance examination for the higher schools.

Algebra. 1. If m arithmetic means be inserted between two numbers a and b, it is required to find the rth term counting from a on.

2. A certain man leased land for 144 yen, and keeping 18 units of it for his own cultivation, he sublet the rest to a third man at a gain of 0.20 yen per unit of land. Then the amount he receives just paid his own rent. Required the number of units of land leased by him.

3. Given the relations $\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{cy+bx}{a-b}$, prove that $(a+b+c)(x+y+z)=ax+by+cz$.

4. Determine the values of p, q, r such that the coefficients of x^5, x^3, and x shall vanish in the expansion of $(x^3-px^2+qx-r)(px^3+x^2+5x+7)$.

Geomstry. 1. If from a point P outside a circle whose center is C two tangents PA and PB are drawn to the circle, and if any chord MN is drawn through the middle point of the chord AB, it is required to prove that the four points P, C, M, N are concyclic and that the line joining the point P and the center C bisects the angle MPN.

2. Prove that the area of a quadrilateral is constant, if the lengths of its two diagonals and the angle included by them are constant.

3. There is a solid angle with three faces, whose diedral angles are each right angles. If the solid angle be cut by a plane, the orthocenter of the triangle formed by the intersections of the cutting plane and the faces of the solid angle is the same point as the foot of the normal dropped from the vertex of the solid angle to the plane.

Trigonometry. This examination included also questions in trigonometry which are not here given.

Since the examinations are held for the purpose of making selections from among the graduates of middle schools, they ought to be in accordance with the subjects taught in these schools. But we are told that sometimes little attention is paid to this. As a rule, emphasis is laid on arithmetic, though nothing in this line has appeared in the above problems for entrance to higher schools. As the subject is taught only in the first years of the middle school, those who go to the examination rarely remember correctly what they have learned so long before. Therefore there are schools where arithmetic is repeated in the highest class or the fifth year, which proves a useless waste of time and labor for the students. For what need is there of using the purely arithmetical knowledge for those who are able to treat the same more simply by the aid of algebra?

Some time ago R. Fujisawa powerfully and ably maintained that arithmetic is the most difficult subject to teach among the various branches of mathematics, and consequently that most attention should be paid to it. This reasonable claim was unfortunately misunderstood as if meant to lay more stress on the subject of arithmetic than on algebra. The result was unfortunate, and we hope the abuse will soon be removed.

In the common school boys and girls are taught alike, but boys only are admitted to the middle school. Corresponding to the latter, however, there are girls' high schools, whose courses are not unique, consisting of three, four, or five years according to circumstances. The girls mostly finish their education in these schools, which are open to the graduates of the ordinary common school. The branches of mathematics taught in the girls' high school are, arithmetic, algebra, and geometry,—all the lessons being simpler than in the middle school. Sometimes algebra or geomety is withdrawn, and solid geometry is never given.

In a certain one of the girls' high schools mathematics is taught two hours per week to all the classes, as follows:

First year: Whole numbers and decimals, various measures, fractions.

Second year: Whole numbers, fractions and decimals, ratio and proportion, percentage.

Third year: Ratio and proportion, percentage, extraction of square root.

Fourth year: Elementary algebra.

Fifth year: Elementary geometry.

The normal school is a school of almost the same grade as the middle school and the girls' high school, and it is here that the teachers of common schools are trained. Most of the normal schools are so constituted that both boys and girls are admitted, although they are taught in different classes; but there are some schools established for boys or girls only. A program for the lessons in mathematics of the normal school was issued recently by the Department of Education. In it we find the interrelation of the various branches of the science emphasized, and the whole has been made more practical, certainly no small advancement in the teaching of mathematics. We hope that the program for the middle school will be arranged with the same aim.

The Higher Normal Schools, which are located in Tokyo and Hiroshima, are schools where the male teachers of the middle class schools are taught. The graduates of the normal school or of the middle school are admitted, and the preliminary course consists of one year, and the main course of three years. The preliminary course is the same for all students of the school, and the mathematics consists of arithmetic, algebra, and geometry, each two hours per week. These lessons are reviews of what has been given in the middle school or in the normal school.

In the main course there is a department for mathematics, physics, and chemistry. The teachers of mathematics are trained here. The department is subdivided into two different classes,—in the first of which mathematics and physics are mainly taught, while in the other physics and chemistry are emphasized.

The subjects of mathematics taught in the first class are as follows:

First year: Algebra, two hours, including series, continued fractions and determinants. These are treated a little more concisely than in C. Smith's *Treatise on Algebra.*

Geometry, two hours, comprising the advanced part of elementary geometry, together with modern geometry. Also trigonometry, two hours.

Second year: Theory of equations, two hours; analytic geometry, four hours, together with seminary exercises.

Third year: The calculus, six hours; solid analytic geometry and dynamics.

In the second class mathematics is taught in a more practical way, its lessons ending in the second year.

F. Sembon and T. Hayashi are the chief professors of the Tokyo Higher Normal School. T. Takehashi teaches in the Hiroshima School.

There are two other schools of the same nature for young women, one in Tokyo, and the other at Nara. In the Tokyo school the students are taught in three separate departments,—letters, science, and arts, each lasting for four years. Mathematics is taught in the science department, and the subjects are arithmetic, algebra, geometry with geometrical drawing, trigonometry, and the teaching of mathematics. These lessons are distributed among the four years, four hours per week in the first year, and three hours in other years.

In the Nara school the science department is subdivided into two courses, in one of which mathematics, physics, and chemistry are taught for the main subjects, and in the other mainly history and mathematics. The same subdivision is to be adopted in the Tokyo school in the coming academic year. The professor of mathematics in the Tokyo Female Higher Normal School is L. Mori.

Besides the main courses in these normal schools sometimes special courses are opened, where the training will end in two or three years.

Further, there is at Sendai a Temporary School for Training Mathematical Teachers. The course extends through two years. K. Hakii is the professor.

Teachers of middle class schools are also granted certificates by special examinations held by the Department of Education. Graduates of middle class schools only are permitted to apply for them. They are divided into two parts, preliminary and final. The former is held by the local governments and the latter by the Department itself. Only those who have been successful in the preliminary examination can take the latter. In mathematics the knowledge of arithmetic, algebra, and geometry is tested in the preliminary examination, and the first part of the final examination is also devoted to the same subjects. Only those who pass the earlier examinations are admitted successively to the examinations in trigonometry, analytic geometry, and the calculus. A terchers' certificate is granted to those who have succeeded in arithmetic, algebra, and geometry. These examinations are the same for men and women, except that the candidates for teachers of female schools are permitted to answer a smaller number of the questions. A few only go up to the calculus.

Those who take these examinations come from various quarters, but for mathematics the Physics School in Tokyo is the main source of supply. This is a private school. The classes extend through three years or six semesters. In the first two years all the students are taught together, while in the last year, the courses are subdivided, the first in mathematics and the other in physical sciences. Mathematics is taught there in the first two years from arithmetic and geometry up to the calculus, and in the third year some more advanced topics of the same subjects are taught. To speak more definitely, these subjects are as follows:

First year: Arithmetic, including theory of prime numbers, etc.;

elementary algebra, including quadratic trinomials, determinants, etc.; elementary geometry, elementary trigonometry.

Second year: Elements of analytic geometry, plane and solid; spherical trigonometry; differential and integral calculus, including algebraic analysis and differential equations.

Third year: Arithmetic and algebra, or theory of numbers and theory of equations, etc.; modern geometry, analytical trigonometry, differential and integral calculus.

Besides these, physics, chemistry, astronomy, and surveying are taught.

The teachers in mathematics are S. Hayashi, Kanazawa, Kariya, Kuniyeda, Mimori, Noguchi, Ogura, Sakuma, Sakurai, Sawayama, Sembon, Terao, and Yasuda.

In the teachers' college of the Waseda University, Tokyo, which is a private school, mathematics is taught in the mathematics department and also in the physics and chemisty departments. In the first of these departments the subjects taught are as follows: Arithmetic, algebra, geometry, trigonometry, which are distributed throughout the whole of three years. Plane analytic geometry in the first year, and solid geometry in the second year. Differential calculus in the second year, and integral calculus including differential equations in the third year.

In the second of the departments, arithmetic, algebra, geometry, trigonometry, and analytic geometry are taught in the first year; solid analytic geometry, and the differential calculus in the second year; and integral calculus in the third year.

The higher schools, eight in number, are preparatory to the Imperial Universities, and are subdivided into three departments. Of these the second department is preparatory to the colleges of science, engineering and agriculture. There mathematics is taught as follows:

First year: Algebra, three hours; and plane trigonometry, two hours.

Second year: Conic sections, three hours; and theory of equations, one hour.

Third year: Calculus, four hours; and dynamics, two hours.

In these subjects the following works are used as text-books or rather as books of reference: C. Smith's *Treatise on Algebra*, subsequent to the chapter on combinations; Todhunter's *Plane Trigonometry;* Puckle's *Conic Sections;* Aldis's *Solid Geometry* (about one-fourth of the book); Burnside and Panton's *Theory of Equations* (the first half); Williamson's treatises on differential and integral calculus, or those of Todhunter; McGregor's *Dynamics.*

Mathematics is not taught to the agricultural students in the third year.

The higher schools are to be reorganized in the near future by the Department of Education although the plan proposed is contrary to pub-

lic opinion. According to the plan the schools will be made institutions where higher general education is given independently of the preparatory work for entrance to the universities, although the graduates are to be received in the latter as before. In that case the courses of instruction will be largely altered, the subject of mathematics being not excepted. The consequence will certainly be a great change in the teaching of the middle school.

The preparatory course of the private Keio University, Tokyo, corresponds to the first department of the higher school, but here mathematics is largely taught with the aim of training the students in the habit of logical thought. This is not generally done elsewhere. The course extends through two years, in the first of which analytic geometry is taught, and in the second the calculus. The chief teacher is S. Fukugawa.

There are two Imperial Universities in Japan, one in Tokyo, and the other in Kyoto. In the mathematics department of the College of Science of Tokyo the following subjects are taught:

First year: Calculus, five hours; analytic geometry, two hours; miscellany in elementary mathematics, two hours; astronomy and method of least squares, three hours; theoretical physics, mathematical seminary, three hours.

Second year: Theory of functions and elliptic functions, three hours; algebraic curves, three hours; differential equations, two hours; theory of numbers and higher algebra, four hours; dynamics, three hours; mathematical seminary and physical experiments.

Third year: Theory of functions and elliptic functions, three hours; higher geometry, two hours; algebra, three hours; differential equations, two hours; miscellany in higher analysis, two hours; dynamics, calculus of variation, three hours; mathematical lectures (optional).

In the departments of astronomy, theoretical physics and experimental physics, mathematics is also taught. In the first year of these departments the following subjects are taught: Calculus; geometry, mathematical exercises, method of least squares. In the second year mechanics, differential equations, etc. In the third year of the first two departments lessons in the theory of functions are given.

In the department of chemistry mathematics is taught for three hours in the first year.

The professors of mathematics are R. Fujisawa, T. Takagi, E. Sakai, and T. Yoshiye, and assistant professor S. Nakagawa.

In some classes of the Engineering College mathematics is being taught somewhat after the practical fashion of John Perry, formerly of the College.

In the Science and Engineering College of Kyoto there is a department of mathematics, wherein are taught the following subjects: Plane and solid analytic goemetry, higher geometry, calculus including differential

equations, theory of functions, higher algebra, theory of numbers, mechanics, theoretical physics, astronomy, physical experiments, and special lectures.

These subjects are taken by the students at convenient times. They are not prescribed for the given years as in the Tokyo University, certainly presenting some advantages over the latter.

The mathematical courses in the departments of engineering are: Integral calculus, three hours for four months; differential equations, two hours for six months; dynamics, two hours for a year; theory of errors, two hours for four months.

The instructors are professors K. Miwa, J. Kawai, assistant professors S. Yoshikawa and T. Wada, and lecturer T. Nishiuchi.

The College of Science of the North-Eastern Imperial University will be opened at Sendai in 1911, with T. Hayashi and M. Fujiwara for professors of mathematics.

In the privately founded universities there is established no colleges of science except in Waseda, the teachers' course of which we have already mentioned. The Tokyo University for Women contains a department for science, which may be mentioned as corresponding to the same in the Normal School for Women, though there will be found some differences in the subjects taught.

The mathematical teaching of some professional schools other than the universities, may also be of some interest.

Technological schools are established in Tokyo, Nagoya, Osaka, Kyoto, and Kumamoto, where the graduates of middle schools are admitted. The mathematics taught in these schools consists of: Algebra, supplementing the knowledge learned in the middle school and including combinations and permutations, series, logarithms, and differential coefficients; solid geometry and important curves; conic sections and solid analytic geometry; the rudiments of the calculus, sufficient to give familiarity with the symbols.

These subjects are taught in some departments only, and in others they are omitted wholly or partly. The head master of mathematics in the Tokyo school is M. Mimori.

The Higher Commercial School of Tokyo provides a one-year preparatory course, and a three-year main course. In the former, mathematics is taught three hours per week. The subjects are: Measures, proportion and interest, problems in equations, series, logarithms and their application, probability, etc. Commercial arithmetic is taught in the first two years of the main course, the professor of mathematics being G. Sawata. There are several other schools of the kind in the country.

Mathematics is not taught in the Military Officers' School but is given in the Military School for Artillerymen and Engineers, and also in the Military College. The first of these schools is the institution where the officers of the army are trained, the other two being intended for the completion of

the officers' education; the Military College produces officers of the staff. The mathematics taught in these schools concludes with the rudiments of conic sections and the calculus.

In the naval schools there are also courses in mathematics. In the Naval Engineering School, at Yokosuka, where three and one-third years are required for graduation, the following subjects are prescribed: Algebra, geometry, trigonometry, analytic geometry, calculus, elementary dynamics, and applied mechanics. A similar plan for the mathematics courses is followed in the Naval School at Yetejima, where all the Japanese naval officers are trained. In the Naval College at Tokyo some courses in mathematics are also given, K. Ashino being professor of mathematics. The teaching of mathematics in the military and naval schools is sometimes criticised for being much behind the times. The mathematics of the Nautical School of Tokyo is almost the same as in the naval schools.

Finally a few words may be said as to the publication of treatises and periodicals which are undoubtedly contributing largely to the improvement of the knowledge of mathematical studies. Although there is no work worthy of exceptional mention, yet the most interesting is certainly the series of treatises published by T. Hayashi. Books of advanced nature have begun to appear occasionally. We shall not however go into details with respect to Japanese manuals.

Among the mathematical journals published in Japan we may mention the *Journal of the Tokyo Physics School*, the *Mathematical Club*, the *Mathemathecal Journal*, the *Mathematical World*, the *XY*, etc. These are, with the exception of the first named, mostly read by the pupils and graduates of middle class schools. The proceedings of the Tokyo Mathematico-Physical Society contains original contributions, mostly composed in European languages. If one desires to know about the progress of mathematics recently made in Japan, he is requested to consult the mathematical journals. The contributors of mathematical essays may be mentioned: Endo, Fujisawa, Fujiwara, Fukuzawa, Hayashi, Kaba, Kaibara, Kariya, Kato, Kikuchi, Kimura, Kitao, Kubota, Kumamoto, Mikami, Miwa, Miwada, Mizuhara, Motoda, Naito, Nakagawa, Ogura, Sudo, Sakai, Sawada, Sawayama, Takagi, Terao, Tsuruta, Yoshiye.

The essays of these authors concern various branches of mathematics, some being historical writings.

The Journal of the College of Science, Tokyo, contains also some articles relating to mathematies, the authors being Fujisawa, Nakagawa, Sakai, Sudo, and Takagi.

Besides, the essays contributed by Japanese mathematicians to foreign periodicals, though few in number, are also to be noted. The writings of Fukuzawa and others cannot escape our notice.

The academic degrees are conferred in Japan by the minister of education upon four classes of candidates: (1) those who have successfully

completed their studies in the Graduate Colleges of Imperial Universities, (2) those who have presented their theses, (3) those who have been recommended by the president of the Imperial University from among the professors. T. Takagi is the only mathematician who has ever received his doctor's degree in Japan through the presentation of a thesis. Among the holders of the same degree in the domain of mathematics we may count R. Fujisawa, J. Kawai, D. Kikuchi, the late D. Kitao (learned in mechanics), K. Miwa, and H. Terao (astronomer).

In Japan the publication of articles does not contribute to elevate the author's social position or improve his reputation, and there are many who have never published anything concerning their special studies and yet who have a reputation as learned men and consequently obtain their high positions. This custom will seem very odd to Americans and Europeans, who are full of the spirit of progress. This is certainly a partial reason why the progress of mathematics in Japan has not been as rapid as could be expected or hoped.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

354. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

To find x in the following equation:

$$0.002\{6x-20[(1.05)^x-1]\}=0.012\{21[(1.05)^{x-1}-1]-(x-1)\}.$$

Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md

The equation readily reduces to

$$3x-35[(1.05)^x-1]=0,$$

from which it is easily seen that $x=0$ is one root of the equation. Another root, we find by a process of approximation, to be $x=21.29$ (nearly).

Also solved by V. M. Spunar, S. G. Barton, and the Proposer.

The equation, being transcendental, has an infinite number of roots, real and imaginary. By compounding the graph of the trandscendental part of the equation with the graph of the linear part, it appears that the above are the only two real roots. ED. F.

355. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

Solve the equations:

$$\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}=a_1;$$
$$\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}=a_2;$$
$$\frac{1}{x_3}+\frac{1}{x_4}+\frac{1}{x_5}=a_3;$$
$$\cdots\cdots\cdots$$
$$\frac{1}{x_{n-2}}+\frac{1}{x_{n-1}}+\frac{1}{x_n}=a_{n-2};$$
$$\frac{1}{x_{n-1}}+\frac{1}{x_n}+\frac{1}{x_1}=a_{n-1};$$
$$\frac{1}{x_n}+\frac{1}{x_1}+\frac{1}{x_2}=a_n.$$

Solution by B. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

Considering the system of equations as linear in the unknowns $\frac{1}{x_1}, \frac{1}{x_2}, \ldots, \frac{1}{x_n}$, the values of the unknowns may be readily found by determinants.

We have for $\frac{1}{x_1}$, the expression,

$$\begin{vmatrix} a_1 & 1 & 1 & 0 & 0 & \cdots & 0 \\ a_2 & 1 & 1 & 1 & 0 & \cdots & 0 \\ a_3 & 0 & 1 & 1 & 1 & \cdots & 0 \\ \cdots & & & & & & \\ \cdots & & & & & & \\ \cdots & & & & 0 & 0 & \\ \cdots & & & & 1 & 1 & 0 \\ \cdots & & & & 1 & 1 & 1 \\ \cdots & & & & 1 & 1 & 1 \\ a_{n-1} & 0 & \cdots & & 1 & 1 & 1 \\ a_n & 1 & \cdots & & 0 & 0 & 1 \end{vmatrix} \div \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 1 & 1 & \cdots & 0 \\ 0 & 0 & 0 & 1 & 1 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \cdots & & & & & & \\ \cdots & & & & & & \\ \cdots & & & & & 1 & 0 \\ \cdots & & & & & 1 & 1 \\ 1 & 0 & 0 & \cdots & & 1 & 1 \\ 1 & 1 & 0 & \cdots & & 0 & 1 \end{vmatrix}$$

That is, $\frac{1}{x_1}=\frac{1}{3}\Delta$, where Δ is the dividend of the above expression, the divisor being equal to 3. When $n\equiv 0 \pmod 3$, the a's are not independent, and the values of the unknowns, $\frac{1}{x_1}$, $\frac{1}{x_2}$, ..., $\frac{1}{x_n}$, are not definite.

When $n\equiv 1 \pmod 3$, the numerator of $1/x_1$ is

$$a_1-2a_2+a_3+a_4-2a_5+a_6+a_7-2a_8+a_9+a_{10}-2a_{11}+\cdots$$

$$=\sum_{i=0}^{i=\frac{1}{3}(n-1)} a_{3i+1}+\sum_{j=1}^{j=\frac{1}{3}(n-1)} a_{3j}-2\sum_{k=0}^{k=I[\frac{1}{3}(n-2)]} a_{3k+2},$$

where $I\left(\frac{n-2}{3}\right)$ means the integral part of $\left(\frac{n-2}{3}\right)$.

The values of $\frac{1}{x_2}$, $\frac{1}{x_3}$, ..., $\frac{1}{x_n}$, may be obtained from that of $\frac{1}{x_1}$ by a cyclic permutation of the coefficients with their proper signs.

When $n\equiv 2 \pmod 3$, the value of the numerator of $1/x_1$ is

$$2a_1-a_2-a_3+2a_4-a_5-a_6+2a_7-a_8-a_9+2a_{10}-\cdots$$

$$=2\sum_{i=0}^{i=I[\frac{1}{3}(n-1)]} a_{3i+1}-\sum_{j=0}^{j=\frac{1}{3}(n-2)} a_{3j+2}-\sum_{k=1}^{k=\frac{1}{3}(n-2)} a_{3k}$$

and the numerators of the other unknowns, $\frac{1}{x_2}$, $\frac{1}{x_3}$, ..., $\frac{1}{x_n}$, may be obtained from that of $1/x_1$, by a cyclic permutation of the coefficients of the a's with their respective signs.

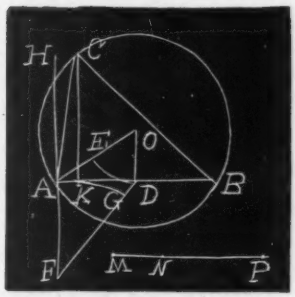
GEOMETRY.

379. Proposed by G. I. HOPKINS, Manchester, N. H.

Construct the triangle, having given base, vertical angle, and ratio of its altitude to difference of other two sides.

Solution by A. H. HOLMES, Brunswick, Maine.

Describe a circle, center O, in which the chord AB (=given base) will subtend the given angle. Draw OD perpendicular to AB, and draw OB. With OD as a radius and O as a center, describe an arc cutting OB in E. Let the divisions MN and NP of the line MP represent the ratio of the difference of the sides of the triangle to the perpendicular. Construct the fourth proportional to MN, NP, and BE, and erect this line, BF, perpendicular to AB at the point B. Draw DF. With F as a center, and FB as a radius, describe an arc cutting FD in G. Find fourth proportional to MN, NP, and $2DG$. Erect this line, BH, perpendicular to AB at point B. Draw HC parallel to AB and cutting circumference in C. Draw AC and BC. Then ABC will be the required triangle, which is shown as follows: $AB=2a$; r, the radius, $=\frac{a}{\cos C}$. Put $x+y$ and $x-y$ for the sides, and p for the ratio of difference of sides to perpendicular. Then we have the proportion, $2r:x+y=x-y:2py$.



$\therefore x^2=y^2+2pry\ldots(1)$.

Also, $\cos C=\frac{(x+y)^2+(x-y)^2-4a^2}{2(x+y)(x-y)}$.

$\therefore (1-\cos C)x^2+(1+\cos C)y^2=2a^2\ldots(2)$.

Eliminating x^2 from (1) and (2), $y^2+2pr(1-\cos C)y=a^2$.

$\therefore y=[a^2+p^2r^2(1-\cos C)^2]-pr(1-\cos C)$.

$\therefore$ Perpendicular $CK=2py=2p\{\sqrt{[a^2+p^2r^2(1-\cos C)^2]}-pr(1-\cos C)\}$.

Also solved by V. M. Spunar.

380. Proposed by W. J. GREENSTREET, A. M., Stroud, England.

$A\ B\ C\ D$ is a quadrilateral, sides in order a, b, c, d, and $B+D=\theta$. Express the diagonals in terms of a, b, c, d, θ.

Solution by A. H. HOLMES, Brunswick, Maine.

Let $ABCD$ be the rhombus, in which $AB=a$, $BC=b$, $CD=c$, $DA=d$. Put $x=AC$, $y=BD$, and $\angle ABC=\psi$. Then $\angle ADC=\theta-\psi$.

In $\triangle ABC$ we have, $\cos\psi=\frac{a^2+b^2-x^2}{2ab}$, and in $\triangle ADC$, $\cos(\theta-\psi)=$

$\frac{c^2+d^2-x^2}{2cd}$. Put $\frac{a^2+b^2}{2ab}=m$, $\frac{1}{2ab}=n$, $\frac{c^2+d^2}{2cd}=p$, and $\frac{1}{2cd}=q$. We find, on eliminating ψ, and reducing,

$$x^4-2\left[\frac{mn-(mq+np)\cos\theta+pq}{n^2-2nq\cos\theta+q^2}\right]x^2=-\frac{m^2-2mp\cos\theta+p^2-\sin^2\theta}{n^2-2nq\cos\theta+q^2}.$$

$$\therefore x=\sqrt{\frac{mn-(mq+np)\cos\theta+pq\pm\sin\theta\sqrt{[n^2-2nq\cos\theta+q^2-(mq-np)^2]}}{n^2-2nq\cos\theta+q^2}}$$

Replacing in this, for m, n, p, and q their equivalents in a, b, c, and d, we have,

$$x=\sqrt{\frac{a^2b^2(c^2+d^2)-abcd(a^2+b^2+c^2+d^2)\cos\theta+c^2d^2(a^2+b^2)\ *\pm abcd\sin\theta\sqrt{[4(a^2b^2-2abcd\cos\theta+c^2d^2)-(a^2+b^2-c^2-d^2)^2]}}{\sqrt{(a^2b^2-2abcd\cos\theta+c^2d^2)}}}$$

For y, we must put b for d in value for x. Also, $-\sin\theta$ for $\sin\theta$, since $\sin(360^\circ-\theta)=-\sin\theta$. Then

$$y=\sqrt{\frac{a^2d^2(b^2+c^2)-abcd(a^2+b^2+c^2+d^2)\cos\theta+b^2c^2(a^2+d^2)\ *\mp abcd\sin\theta\sqrt{[4(a^2d^2-2abcd\cos\theta+b^2c^2-(a^2-b^2-c^2+d^2)^2]}}{\sqrt{(a^2d^2-2abcd\cos\theta+b^2c^2)}}}$$

When $ABC+ADC>180^\circ$, $AC<BD$, and the plus sign is to be used before $abcd\sin\theta$ for x, and the minus sign before the same for y. This is, of course, reversed when $ABC+ADC<180^\circ$.

381. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find the number of diagonals of a *complete* polygon of n sides.

I. Solution by J. OWEN MAHONEY, B. E., M. Sc., Central High School, Dallas, Texas.

An incomplete polygon of n sides has $\frac{n}{2}(n-3)$ diagonals. The complete polygon of n sides has $\frac{n}{2}(n-1)$ vertices or $\frac{n}{2}(n-3)$ more than the incomplete polygon of n sides. Hence, the number of diagonals of the complete figure is,

$$\frac{n}{2}(n-3)+\frac{\left[\frac{n}{2}(n-3)\right]\left[\frac{n}{2}(n-3)-1\right]}{2}$$

$$=\frac{n}{2}(n-3)+\frac{n(n-3)\,(n^2-3n-2)}{8}=\frac{n(n-1)\,(n-2)(n-3)}{8}.$$

II. Solution by the PROPOSER.

A *complete* polygon of n sides is formed by n intersecting lines, and the first question is, how many lines will join the points of intersection of n intersecting lines. n lines can intersect in $\frac{n(n-1)}{2}$ points. From each of these points there can be drawn the other $\frac{n(n-1)}{2}-1$ lines. Therefore, from $\frac{n(n-1)}{2}$ points, $\frac{n(n-1)}{2}\left[\frac{n(n-1)}{2}-1\right]$, of which, evidently, only half can be taken; consequently, the required number of lines is $\frac{1}{8}(n+1)n(n-1)(n-2)$. Since each line drawn from $\frac{n(n-1)}{2}$ points cuts $n-2$ lines, we must subtract $\frac{n(n-1)}{2}(n-2)$ from the last result, in order to obtain the number of diagonals of a complete polygon of n sides. Therefore the required number of diagonals is

$$\begin{aligned}&\tfrac{1}{8}(n+1)n(n-1)(n-2)-\tfrac{1}{2}n(n-1)(n-2)\\&=\tfrac{1}{8}n(n-1)(n-2)(n-3),\end{aligned}$$

or, more briefly, $3\binom{n}{4}$.

Also solved by Jeannette Brooks and A. H. Holmes.

CALCULUS.

382. Proposed by PROFESSOR L. E. DICKSON, Ph. D., The University of Chicago.

Find an algebraic integral of

$$LCy''+[18L(s-x^2)-2kC]y'+[L(\tfrac{2}{3}k-10x)-8k(s-x^2)]y=0,$$

where $L=2kx+3s-k^2$, $C=12sx-4x^3-t$, $k^4-6sk^2-tk-3s^2=0$. The roots of the determining equation are 0, $-\frac{1}{2}$ for a root of $C=0$; 0, 2 for a root of $L=0$; and 1 and $\frac{3}{2}$ at infinity.

No solution of this problem has been received.

383. Proposed by C. N. SCHMALL, New York City.

If $A=\int_0^1 \frac{dx}{\sqrt{(1-x^4)}}$, express A in terms of Gamma-functions. [Part of ex. 39, p. 474, Bromwich's *Theory of Infinite Series.*]

I. Solution by JOSEPH A. NYBERG, Student, University of Chicago.

In the integral

$$u=\int_0^1 x^{\lambda-1}(1-x^{\mu})^{\nu-1}dx \tag{1}$$

make the substitution, $x^{\mu}=y$. Then

$$dx=\frac{1}{\mu}y^{(1/\mu)-1}dy, \quad \begin{cases} x=0, \ldots, 1 \\ y=0, \ldots, 1 \end{cases}$$

Therefore,

$$u=\frac{1}{\mu}\int_0^1 y^{(\lambda/\mu)-1}(1-y)^{\nu-1}dy\equiv\frac{1}{\mu}B\left(\frac{\lambda}{\mu},\nu\right) \tag{2}$$

by the definition of the beta function. Using the formula,

$$B(a,b)=\frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} \tag{3}$$

we get

$$u=\frac{1}{\mu}\frac{\Gamma(\lambda/\mu)\,\Gamma(\nu)}{\Gamma[(\lambda/\mu)+\nu]}. \tag{4}$$

(4) is the solution of (1) in terms of gamma functions.

On putting $\nu=\frac{1}{2}$, $\lambda=1$, $\mu=4$, we have

$$\int_0^1\frac{dx}{\sqrt{(1-x^4)}}=\frac{1}{4}\frac{\Gamma(\frac{1}{4})\,\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4})}.$$

Proofs of the formula,

$$B(a,b)=\frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}$$

are found in Byerly's *Integral Calculus*, p. 114; in an article by A. Pringsheim in *Mathematische Annalen*, Band 31 (1888), p. 480; and in N. Nielsen's *Hankbuch der Theorie der Gamma Funktion*, p. 148.

II. Solution by FRANCIS RUST, C. E., Pittsburg, Pa.

Let $x^2=\sin\phi$. Then $dx=\frac{\cos\phi\,d\phi}{2\sqrt{(\sin\phi)}}$. Hence,

$$A=\tfrac{1}{2}\int_0^{\frac{1}{2}\pi}\sin^{-\frac{1}{2}}\phi\,\cos\phi\,d\phi=\tfrac{1}{4}B(\tfrac{1}{4},\,\tfrac{1}{2})=\tfrac{1}{4}\frac{\Gamma(\frac{1}{4})\,\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4})}=\tfrac{1}{4}\sqrt{(\pi)}\cdot\frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}.$$

In elliptic integrals, using Legendre's notation, $A=\frac{1}{2}\sqrt{(2)}F'(\frac{1}{2}\sqrt{2})=1.311029$.

Also solved similarly by James A. Whitted, and the Proposer.

304. Proposed by H. C. FEEMSTER, York College, York, Neb.

Reduce $axyp^2+(x^2-ay^2-b)p-xy=0$ to Clairaut's form, and hence solve the equation.

Solution by OLLINE CRUICKSHANKS, Bowling Green, Ky., and S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, New York.

Expanding and factoring,

$$apy(px-y)+x(px-y)=bp,\quad (apy+x)(px-y)=bp. \tag{I}$$

Let $x^2=u$ and $y^2=v$. $\therefore 2xdx=du$ and $2ydy=dv$.

$$p=\frac{dy}{dx}=\frac{x}{y}\frac{dv}{du}.$$

Substituting this value of p in (I),

$$\left(\frac{x^2}{y}\frac{dv}{du}-y\right)\left(\frac{axy}{y}\frac{dv}{du}+x\right)=\frac{bx}{y}\frac{dv}{du}. \tag{II}$$

Multiplying (II) by y/x,

$$\left(x^2\frac{dv}{du}-y^2\right)\left(a\frac{dv}{du}+1\right)=b\frac{dv}{du}. \tag{III}$$

Since $x^2=u$ and $y^2=v$, (III) becomes

$$(u\frac{dv}{du}-v)(a\frac{dv}{du}+1)=b\frac{dv}{du}. \tag{IV}$$

$$\therefore v=u\frac{dv}{du}-\frac{b\frac{dv}{du}}{a\frac{dv}{du}+1}. \tag{V}$$

$\therefore v=uc-\dfrac{bc}{ac+1}$; but $v=y^2$ and $u=x^2$. $\therefore y^2=cx^2-\dfrac{bc}{ac+1}$.

Also solved by A. H. Holmes, who substitutes for p, xz/y, and then differentiates the resulting equation.

305. Proposed by C. N. SCHMALL, New York City.

Prove $\int_{\beta}^{x} \frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}} = 2\cos^{-1}\sqrt{\frac{\alpha-x}{\alpha-\beta}}$. [Edwards' *Integral Calculus for Beginners*, p. 84, ex. 4.] Does this result hold when the upper limit is changed from x to α?

Solution by A. M. HARDING, Associate Professor of Mathematics, University of Arkansas.

Put $x=\beta\cos^2\theta+\alpha\sin^2\theta$, *i. e.*, $\theta=\cos^{-1}\sqrt{\frac{\alpha-x}{\alpha-\beta}}$; then $\alpha-x=(\alpha-\beta)\cos^2\theta$, $x-\beta=(\alpha-\beta)\sin^2\theta$, $dx=(\alpha-\beta)\sin 2\theta\, d\theta$.

Hence, $\int_{\beta}^{x} \frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}} = \int_{0}^{\theta} \frac{(\alpha-\beta)\sin 2\theta\, d\theta}{(\alpha-\beta)\sin\theta\cos\theta}$

$$= 2\int_{0}^{\theta} d\theta = 2\theta = 2\cos^{-1}\sqrt{\frac{\alpha-x}{\alpha-\beta}}.$$

When the upper limit is α, we have

$$\int_{\beta}^{\alpha} \frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}} = \operatorname*{Lim}_{\varepsilon \doteq 0} \int_{\beta}^{\alpha-\varepsilon} \frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}}$$

$$= \operatorname*{Lim}_{\varepsilon \doteq 0} 2\cos^{-1}\sqrt{\frac{\alpha-(\alpha-\varepsilon)}{\alpha-\beta}} = 2.\tfrac{1}{2}\pi = \pi.$$

Also solved in various ways by C. E. Rust, A. H. Holmes, V. M. Spunar, and the Proposer.

MECHANICS.

253 (Incorrectly numbered 354). Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

On a smooth horizontal plane are touching each other two balls, equal, uniform, inelastic, spherical, radius x, mass M. A third uniform, smooth, elastic ball, radius y, mass m, is placed with its centre vertically above the point of contact of those balls. Its radius may vary as long as its mass remains m. When is the velocity of each inelastic ball after impact a maximum?

No solution of this problem has been received.

354 (Incorrectly numbered 355). Proposed by the late G. B. M. ZERR, Ph. D.

Find the current given by a battery $\frac{1}{2}n(n+1)$ cells arranged in n rows as follows: First row, 1 cell; second row, 2 cells; third row, 3 cells; nth row, n cells. The electromotive force of each cell is E and its resistance r, while the resistances of the wires joining the positive and negative ends of the rows to the poles are as follows: First row, nr; second row, $(n-1)r$; third row, $(n-2)r$, ..., $(n-2)$nd row, $3r$; $(n-1)$st row, $2r$; nth row, r. The resistance of the two wires joining the poles to complete the circuit is r for each.

Solution by V. M. SPUNAR, C. E., Chicago, Ill.

If the cells in the mth row are connected in series, the current in that row is, by Ohm's law,

$$C_m=\frac{mE}{mr+R_m},$$

where R_m is the external resistance$=2r(n-m+1)$, by the conditions of the problem. Hence, the total current is

$$C=\sum_{m=1}^{m=n}\frac{mE}{mr+2r(n-m+1)+2r}=\frac{E}{r}\sum_{m=1}^{m=n}\frac{m}{2(n+2)-m}$$

$$=\frac{E}{r}\sum_{m=1}^{m=n}\left[-1+\frac{2(n+2)}{2(n+2)-m}\right]=\frac{E}{r}\left[-n+2(n+2)\sum_{m=1}^{m=n}\frac{1}{2(n+2)-m}\right]$$

$$=\frac{E}{r}\left[2(n+2)\left(\frac{1}{2n+3}+\frac{1}{2n+2}+\frac{1}{2n+1}+\ldots+\frac{1}{n+4}\right)-n\right]$$

$$=\frac{E}{r}\left[2(n+2)\,(S_{2n+3}-S_{n+3})-n\right],\quad\text{where } S_p=\tfrac{1}{1}+\tfrac{1}{2}+\tfrac{1}{3}+\ldots+\frac{1}{p}.$$

But $S_{2n+3}-S_{n+3}=\log(2n+3)-\log(n+3)$

$$+\frac{1}{2(2n+3)}-\frac{1}{2(n+3)}-\frac{1}{12(2n+3)^2}+\frac{1}{12(n+3)^2}\cdots,$$

(see MONTHLY, Vol. XVI, p. 166)

$$=\log\left(\frac{2n+3}{n+3}\right)+\tfrac{1}{2}\cdot\frac{n}{(n+3)\,(2n+3)}+\tfrac{1}{4}\cdot\frac{n(n+2)}{(n+3)^2\,(2n+3)^2}+\ldots$$

When $n\doteq\infty$, $S_{2n+3}-S_{n+3}\doteq\log 2$.

NOTE. Contributors are asked to send in solutions of problems 255, 256, 257, 258, and 259. 255 is on page 247, Vol. XVII.

AVERAGE AND PROBABILITY.

203. Proposed by J. EDWARD SANDERS, Weather Bureau, Columbus, Ohio.

Find the average length of a hole at random through a cone.

No solution of this problem has been received.

204. Proposed by F. P. MATZ, Reading, Pa.

On a random chord in a circle two points are taken at random. What is the chance a second chord drawn at random will pass between the two points?

Solution by the late G. B. M. ZERR, Ph. D.

Let M, N be the two random points on the random chord AB; O, the center of the circle; $OA=OB=r$; $AM=y$; $MN=x$; $\angle AOB=2\theta$; μ the angle AB makes with some fixed line. Then $AB=2r\sin\theta$; an element of the circle at M is $r\sin\theta\, d\theta\, dy$; at N, $d\mu\, xdx$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of μ, 0 and 2π; of y, 0 and $2r\sin\theta$; of x, 0 and y. If the second random chord intersects the first, the chance that it passes between the points is $x/2r\sin\theta$. The chance that the second chord intersects the first is $\frac{1}{3}$. Therefore, if p=the chance, we get

$$p=\frac{\frac{1}{3}\int_0^{\frac{1}{2}\pi}\int_0^{2\pi}\int_0^{2r\sin\theta}\int_0^{y} rx^2\sin\theta\, d\theta\, d\mu\, dy\, dx}{\int_0^{\frac{1}{2}\pi}\int_0^{2\pi}\int_0^{2r\sin\theta}\int_0^{y} 2r^2x\sin^2\theta\, d\theta\, d\mu\, dy\, dx}=\frac{1}{6}.$$

Also solved by J. E. Sanders, who obtains 3/16 as a result.

MISCELLANEOUS.

177. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Sum the infinite series:

(a) $\sin x+nx\cos x-\frac{n^2x^2}{2!}\sin x-\frac{n^3x^3}{3!}\cos x+\frac{n^4x^4}{4!}\sin x+\ldots,$

(b) $\cos x-nx\sin x-\frac{n^2x^2}{2!}\cos x+\frac{n^3x^3}{3!}\sin x+\frac{n^4x^4}{4!}\cos x\ldots$

Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md., and G. B. M. ZERR, Ph. D.

(a) Arranging, we get

$$\sin x\left[1-\frac{n^2x^2}{2!}+\frac{n^4x^4}{4!}\ldots\right]+\cos x\left[nx-\frac{n^3x^3}{3!}+\frac{n^5x^5}{5!}\ldots\right]$$

$=\sin x\cos nx+\cos x\sin nx=\sin(n+1)x.$

(b) $$\cos x\left[1-\frac{n^2x^2}{2!}+\frac{n^4x^4}{4!}\ldots\right]-\sin x\left[nx-\frac{n^3x^3}{3!}+\frac{n^5x^5}{5!}\ldots\right]$$

$=\cos x\cos nx-\sin x\sin nx=\cos(n+1)x.$

Also solved by Francis Rust and V. M. Spunar.

178. Proposed by V. M. SPUNAR, Mechanical and Civil Engineer, East Pittsburg, Pa.

Find the sum of the series, $\frac{\sin x}{m^2+1}-\frac{2\sin 2x}{m^2+4}+\frac{3\sin 3x}{m^2+9}-\ldots$ to infinity.

Solution by G. B. M. ZERR, Ph. D.

From the general equation in Fourier's Series, we at once derive

$$\frac{\pi}{2}\sin nx=\sin x\int_0^\pi \sin nx\,\sin x\,dx+\sin 2x\int_0^\pi \sin nx\,\sin 2x\,dx$$

$$+\sin 3x\int_0^\pi \sin nx\,\sin 3x\,dx+\ldots$$

But $\int_0^\pi \sin nx\,\sin rx\,dx=\pm\frac{r\sin n\pi}{r^2-n^2}$, according as r is odd or even.

$$\therefore \frac{\pi\sin nx}{2\sin n\pi}=\frac{\sin x}{1-n^2}-\frac{2\sin 2x}{4-n^2}+\frac{3\sin 3x}{9-n^2}-\ldots$$

Let $n=m\sqrt{-1}$.

$$\therefore \frac{\sin x}{m^2+1}-\frac{2\sin 2x}{m^2+4}+\frac{3\sin 3x}{m^2+9}-\ldots=\frac{\pi\sinh mx}{2\sinh m\pi}=\frac{\pi}{2}\cdot\frac{e^{mx}-e^{-mx}}{e^{m\pi}-e^{-m\pi}}.$$

NOTES AND NEWS.

Professor Jose A. Caparo, of Notre Dame University, Notre Dame, Indiana, has been given a year's leave of absence, and will spend most of his time in Cuzco, Peru, South America. F.

Before leaving for California, Editor Slaught was called to New York State by his Alma Mater, Colgate University, to receive the honorary degree of Doctor of Science. He graduated there in 1883, and has been engaged in educational work ever since. The degree of Doctor of Philosophy was conferred upon him by the University of Chicago, in 1898, in which institution he is now an Associate Professor of Mathematics. He is the author, in collaboration with Dr. Lennes, of Columbia University, of a series of text books in mathematics for secondary schools, which are being very extensively adopted throughout the country. We congratulate Dr. Slaught on the honor conferred upon him by his Alma Mater. F.

Editor Slaught is in California attending the meeting of the National Education Association as the representative of the University of Chicago. He will present the report of the National Committee of Fifteen on Geometry at the meeting of the Secondary Department. This committee has been working for two years under the joint auspices of the National Education Association and the American Federation of the Physical and Mathematical Sciences. The report in provisional form was published in the April, May, and June issues of *School Science and Mathematics* and was distributed in the form of reprints to two hundred selected critics for further suggestions and criticisms before the final presentation at San Francisco. The report contains a valuable historical introduction by Professor Florian Cajori, which was initially published in THE MONTHLY for November, 1910. Then follow sections on logical considerations, special courses in geometry, exercises and problems, and finally the syllabus itself, which sets forth a scheme for emphasizing the great basal theorems of geometry, in comparison with those of subsidiary importance, by means of various styles of type. ED. F.

The following doctors and candidates for the doctorate in mathematics from the University of Chicago, have been appointed to positions for 1911-1912, as indicated below:

Professor F. L. Griffin, 1907, of Williams College, to the professorship of mathematics at Reed Institute, Portland, Oregon.

Dr. H. E. Buchanon, 1909, of the University of Wisconsin, to the professorship of mathematics at Carleton College, Northfield, Minnesota.

Dr. E. J. Miles, 1910, of Cornell University, to an instructorship in mathematics at Yale University.

Dr. Anna J. Pell, 1910, to an instructorship in mathematics at Mt. Holyoke College.

Mr. Lloyd Dines to an instructorship in mathematics at Columbia University.

Dr. D. Buchanon, 1911, to an assistant professorship in mathematics at Queen's University, Kingston, Ontario.

Mr. R. E. Root to an instructorship in mathematics at the Univesity of Missouri.

Messrs. Dines, and Root are finishing their work during the present summer quarter. S.

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THE TEACHING OF MATHEMATICS IN SUMMER SESSIONS OF UNIVERSITIES AND NORMAL SCHOOLS.*

By H. E. SLAUGHT, The University of Chicago.

THE SUMMER SESSIONS IN GENERAL.

Previous to the year 1891 there had been no general movement in this country to organize summer courses in connection with the schools, colleges, and universities. Certain summer courses for teachers had been offered at Harvard University as early as 1871, but not until 1891 were any regular courses in mathematics given. In 1891 also certain courses in mathematics and latin were offered in a summer session at the State Normal School at Emporia, Kansas, as a private undertaking of two instructors in the school. In like manner some summer courses were offered in the same year at the University of California, the University of Indiana, and the University of Minnesota. No doubt, complete data would show that similar beginnings were made about this time, or not much later at other institutions. Cornell University made a start in 1892, the University of Missouri and the University of Illinois in 1894, and the University of Texas in 1897. In most of these cases, if not all, the beginnings were made by a few instructors who carried on the work as a semi-private undertaking, depending wholly upon the tuition receipts for their remuneration.

The University of Chicago was the first institution to embody in its constitution a provision for a regular summer session conducted by the permanent faculty under conditions precisely similar to those in the other sessions of the school year. President Harper maintained that to allow a great and costly University plant to stand idle during one quarter of the year was neither good business nor good educational policy. He held that the advantages of college and university training should not be kept forever out of reach of those who were engaged in teaching and who could not afford to

* This paper was prepared on short notice at the request of the American Commissioners of the International Commission on the Teaching of Mathematics. They discovered at the last moment that this phase of work for the preparation of teachers had been overlooked in the assignment of jurisdiction to other committees. As there was not time for an exhaustive investigation, it was decided to gather data from a limited number of representative institutions and to draw such conclusions as might be possible on this basis.